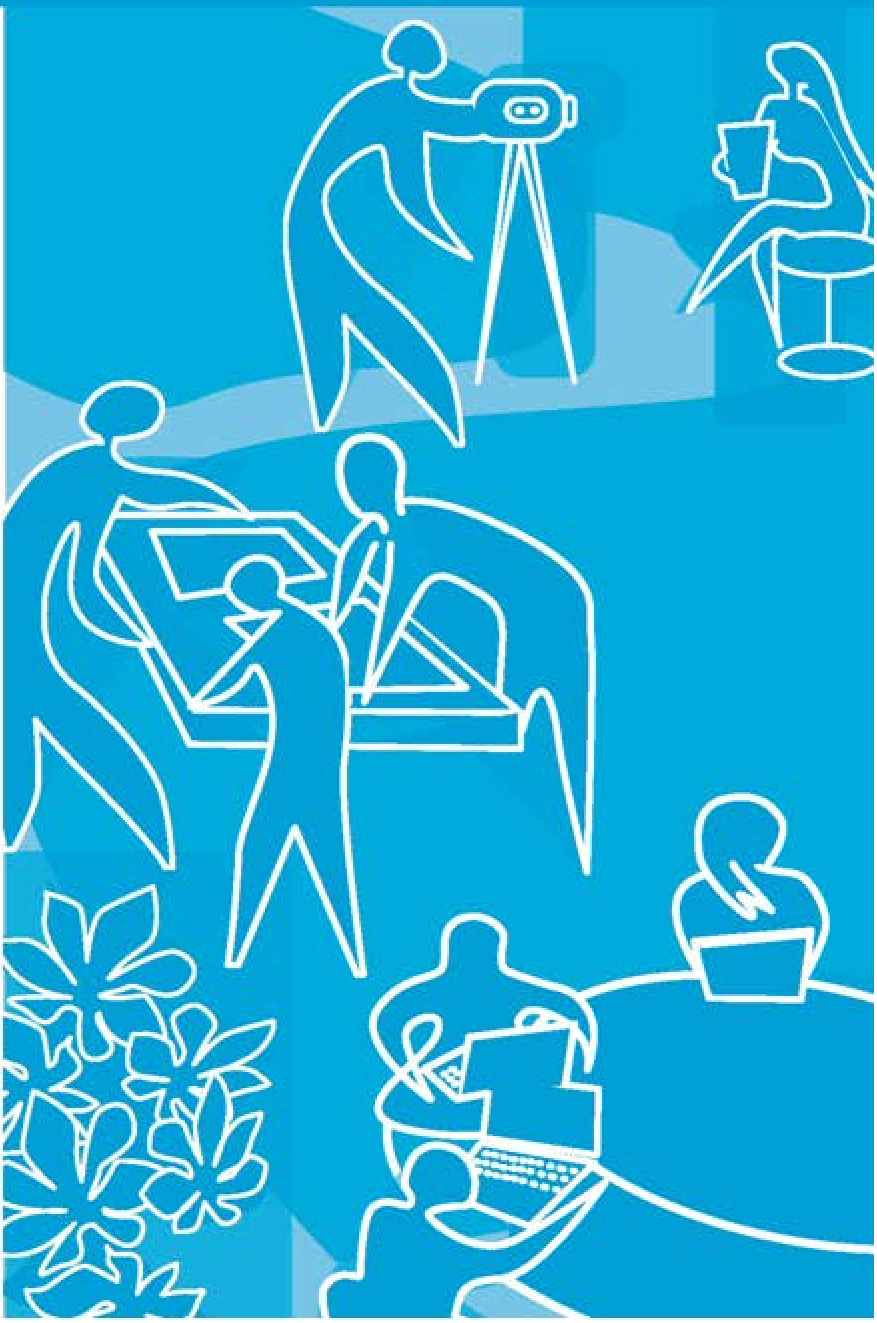
NUMERACY TEACHER TOOLKIT



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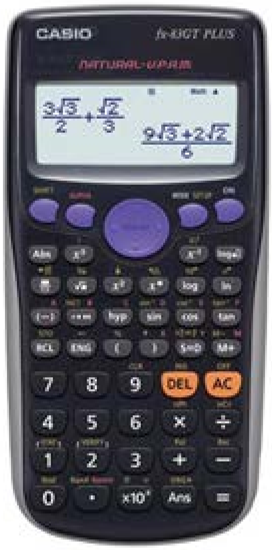
Equipment



This is the Casio FX-83GTPLUS which the school recommends. Pupils are requested to use Casio calculators rather than Texet etc.

The normal pen, pencil, rubber and sharpener are required along with a mathematics set with the minimum pieces…
* Protractor
* Ruler
* Compass

Introduction

The following pages include methods for teaching/using certain skills relevant to Numeracy and/or Mathematics. The aim of this toolkit is to support teachers' and parents' understanding of the current approaches used within the school to teach these skills. A consistency of approach will support student understanding.

The toolkit contains the basic mathematical concepts, equipment used and methods which we would be grateful if you consider when planning the numeracy elements of your lessons. Please however keep in mind **"If it isn't broke don't fix it."** In other words, if the pupils are <u>correctly</u> using a different approach to that which is stated in this document then allow them to continue with that method. Think of the following pages' methods as the *default* approaches. It is important to ask the student how they approach a problem and support this method. At the start of each topic a list of the common mathematical vocabulary associated with that topic is given.

If you require further detail of any of the methods, the Numeracy Coordinator and/or a member of the Mathematics department will be very pleased to liaise with you.

Thank you for your cooperation.

This booklet is adapted from the original written by Liam Donovan, Treorchy Comprehensive School.

Last edit June 2014 by G James with input from the Mathematics department at St Alban's RC High School.

Number

Place Value

Vocabulary: Units, Tens, Hundreds etc. Place Value, Multiply, Divide

i) Place holders

- Place value is important when dealing with numbers. Must pupils will be comfortable between units and thousands.
- Place value is needed to understand size and also helps when multiplying and dividing by 10, 100, 1000 etc.

Units *Tenths* *Hundredths ....*

10000s	1000s	100s	10s	1s	$\frac{1}{10}$s	$\frac{1}{100}$s	$\frac{1}{1000}$s	$\frac{1}{10000}$s

Due to the way people discuss dates *number slang* has been a factor which has caused some pupils to find reading or writing large numbers difficult.

e.g. 2012 as a year would be said "twenty twelve" as opposed to Two Thousand and twelve.

A common mistake pupils make would be for example the number two thousand and forty three written as 200043.

The best way to tackle this problem is to write the digits in the place value holders (like in the diagram above). If you feel your subject would benefit from these please ask and some hard copies could be made.

ii) $\times / \div$ by powers of 10

x 10, 100, 1000 …

When pupils are asked to multiply by a power of 10 they are taught to move the digits in the place value holders to the left (see diagram below) by however many zeros are in the number that you are multiplying by. **Please remember the decimal point doesn't move.**

e.g.

What is 3.1 × 100?

Thousands	Hundreds	Tens	Units	tenths	hundredths	thousandths
	3	1	0			

N.B. the zero is added to the unit's column as any blank column found to the left side of the decimal point must always be filled to support the new place value.

÷ 10, 100, 1000 …

When dividing by powers of ten pupils use the same idea as above in multiplying apart from the fact that now the digits move in the opposite direction (to help aid this idea remind pupils that when dividing you are generally making the numbers smaller). **Please remember that the decimal point does not move.**

What is 510 ÷ 1000?

Thousands	Hundreds	Tens	Units	tenths	hundredths	thousandths
			0	5	1	

The use of the place holder stops pupils getting confused when they use the *take off the noughts* method. In the example above with only one nought in 510 and the fact the pupils think they need to take off 3 noughts they'll often get this type of question wrong with that method.

Addition

Vocabulary: Sum, Total, More.

Written Approach

```
 276
  68 +
 344
 1 1   ←——  carried on values
```

N.B. Place value is vitally important here for pupils that struggle. Get them to write U, T, H etc. above each digit of a number.

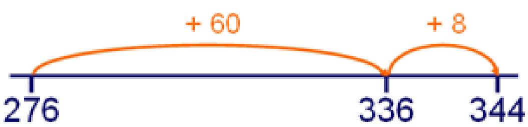
Place Value columns are especially important when numbers include decimals and/or multiple zeros.

```
4500 + 83.51 → 4 5 0 0.00                    4 5 0 0
                 8 3.51 +   instead of       8 3. 5 1 +
```

Alternative Approach

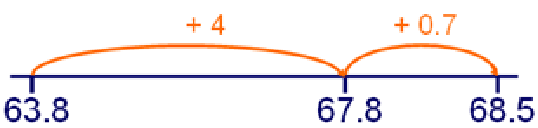
What is 276 + 68?

276 + 60 + 8 = 344



What is 63.8 + 4.7?

63.8 + 4 + 0.7 = 68.5



Subtraction

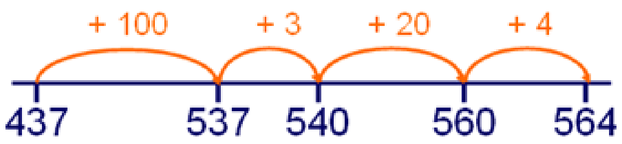
Vocabulary: Difference, Less.

Mental Approach

Subtracting by counting up

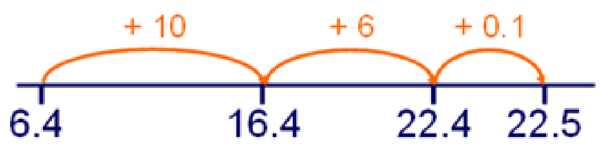
What is 564 – 437?

100 + 3 + 20 + 4 = 127



What is 22.5 – 6.4?

10 + 6 + 0.1 = 16.1



Some pupils should understand how this method is far quicker/easier for subtractions where the numbers are close.

e.g. 24 – 17

Using the 'counting up' approach stops having to remember to 'borrow'.

Also, once pupils are fairly secure with numbers and place value they will find this method easier when the numbers include multiple zeros. The reason being 'borrowing' is once again removed from the steps.

e.g. 5000 – 2899

Here we count +1 (to 2900) +100 (to 3000) +2000 (to 5000)
So final answer is 1+100+2000 = 2101

Written Approach

Here pupils will need to be reminded about "borrowing". However, once again, understanding of place value is vital as pupils need to see that, for example in the problem below, when crossing out the 6 and leaving a 5 they are taking one ten and adding it to the units column. So now we have 5 Hundreds, 5 tens and 14 units which still totals 564. In fact there is no "borrowing" going on at all – it is better described as "reallocating" but pupils may find this word confusing!

e.g.

Solve 564 – 437

Partitioning	500	60	4	–
	400	30	7	
Becomes	500	50	14	–
	400	30	7	
	100	20	7	→ 127

Leads to …

$$\begin{array}{ccc} 5 & {}^{5}\cancel{6} & {}^{1}4 \\ 4 & 3 & 7 - \\ \hline 1 & 2 & 7 \end{array}$$

Multiplication

Vocabulary: Product, Multiply

There are three methods commonly used for multiplication.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Use of the multiplication grid may be helpful

In the below example of the Box Method there are two columns (vertical) as the number 16 has 2 digits. There are two rows (horizontal) as the number 85 has 2 digits.

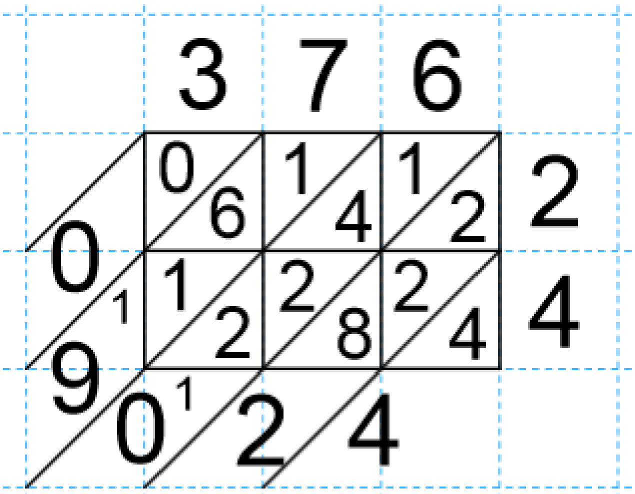
e.g. 16 x 85

X	10	6		
80	800	480	=	1280 +
5	50	6 x 5 30	=	80
				1360

The values inside each inner box are found by **multiplying** the column and row value. Once each inner box is found the final answer is found by **adding** the values together.

The following is an example of the Napier's Bones method. Pupils may refer to this as the Grid Method.

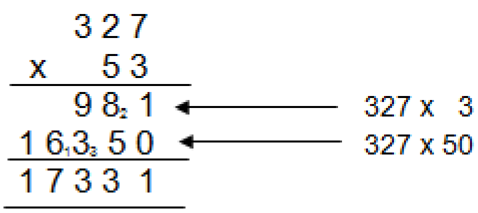
e.g. 376 x 24



If you start in the top right hand corner you can see that as 6 x 2 = 12 the digits 1 and 2 have been inserted either side of the diagonal line. In the case of the top left box 3 x 2 = 6. The tens digit of the number 6 is 0 so 0 goes above the line and 6 underneath. After doing this for each pair of digits you add together numbers along the diagonal, carrying tens digits over where necessary. The answer is then read from left to right as 9 024.

The Traditional Method for written multiplication is still popular.

e.g. 327 x 53



This method depends on pupils remembering to put a zero down first when multiplying by the tens digit.

Division

<u>Vocabulary:</u> Share

To solve all division questions the pupils are encouraged, if number skills are poor, to make a list of the times table of the number they are dividing by.

e.g. $$\begin{array}{r} 0\,1\,6 \\ 15\overline{)\,2^{2}6^{9}0} \end{array}$$ 15, 30, 45, 60, 75, 90 …

The Maths department also teach the traditional long division method like the example below.

$$\begin{array}{r}
3475 \\
25\overline{)\,86894} \\
-75 \\
\hline
118 \\
-100 \\
\hline
189 \\
-175 \\
\hline
1\cancel{4}^{3}\,{}^{1}4 \\
-125 \\
\hline
19
\end{array}$$

Time calculation

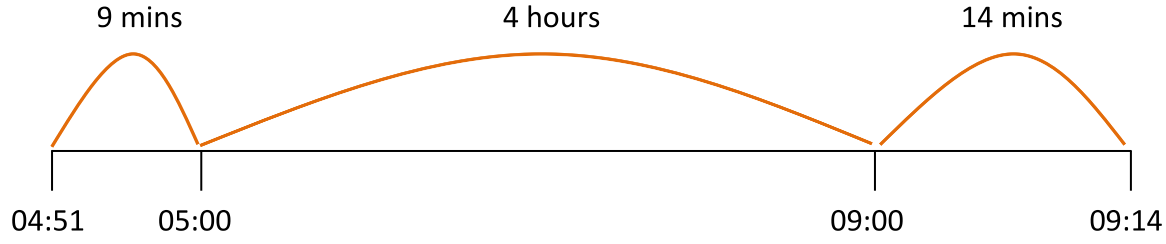
<u>Vocabulary:</u> am, pm, minutes, hours

Never use a calculator for time calculations.

Example:

Jonathan leaves Cumbria at 04:51 and arrives in Cwmbran at 09:14. How long is his journey?

To solve questions such as the above – use a number line.



Total journey is 4 hours 23 minutes.

Distance Charts

<u>Vocabulary: Miles, kilometres, difference, total.</u>

	Aberdeen	Brighton	Carlisle	Exeter
Brighton	540			
Carlisle	212	350		
Exeter	555	166	345	
Glasgow	142	442	92	440

Read these by matching the columns and the rows for the destinations you want to travel between.

E.g. Brighton to Exeter = 166 miles.

N.B. it is worth noting pupils should write miles for miles and not abbreviate to m which is used for metres.

Rounding

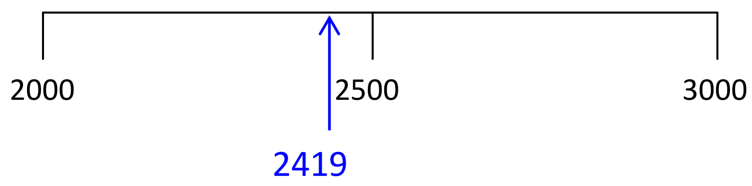
<u>Vocabulary:</u> Whole number (Integer), Decimal Places, Significant Places.

To aid the pupils' memory about how to round here is a ***Rounding Rhyme***.

Find the number look right next door,
Four or less just ignore,
Five or more add one more.

E.g. Round 2419 to the nearest thousand

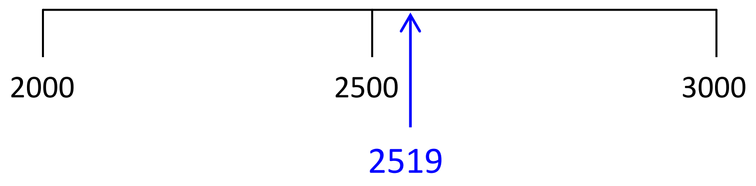
Find the number (in this case the thousand) look right next door (here the digit to the right is a 4)
Four or less (which the digit is) just ignore (so we don't need to increase the 2 thousand to a 3) → answer is 2000



Using a number line with the mid point marked it is possible to visually represent where the number is and therefore if it is closer to 2000 or 3000.

If however the question was round 2519 to the nearest thousand we would end up with…

Four or less (which the digit isn't as this time it is a 5) just ignore
Five or more (which it is) add one more (so the 5 makes the 2 thousand become a 3 thousand)
→ answer is 3000
N.B. all the other place value columns are now zeros.



In this case nearer to 3000.

Some pupils like the idea of *if it's past half way you round up, and if not, you round down*. This method is fine but it is noticed that pupils generally become unsure when they are dealing with decimal place values.

Decimals

Vocabulary: Decimal point, Ascending, Descending.

- Use of the place holders is very important when dealing with decimals.
- When reading decimals each digit is read as a single unit value digit number.

E.g. 0.12 is read as "zero point one two" and **not** "zero point twelve".

WHY?
It is simply understood as a way of emphasising a number's size.

E.g. reading 0.12 as "…point twelve" would lead pupils to think that 0.12 is bigger than 0.8 → because twelve is bigger than eight.

When ordering decimals it is important to use the idea of the place holders. It can help some pupils to add *extra noughts* to some of the decimals to aid their understanding.

E.g. Put 0.3, 0.29, 0.31, 0.309 in ascending order.

U	**.**	**1/10**	**1/100**	**1/1000**
0	.	3	0	0
0	.	2	9	0
0	.	3	1	0
0	.	3	0	1

Tell pupils to order them by looking **down** the place value columns regardless of how many numbers come after them. If there is more than one answer with the same digit in a column move onto the next lower value column until you find the smallest.

Alternatively if still unsure tell pupils to add zeros to make all numbers have the same number of digits (see black zeros in example above). This usually helps them decide on the size of the number.

Percentages

Vocabulary: Proportion, percentage.

Mental Calculation Strategy

- Always start by finding 10%, by dividing by 10 (because 10% is the same as one tenth)
- Then use this answer and halve it to find 5%, halve again to find 2.5%, etc.
- To find 1%, divide by 100.

e.g. Find 10% of 120

120 ÷ 10 = 12

e.g. Find 17.5% of 120

10% = 120 ÷ 10 = 12
5% = 12 ÷ 2 = 6
2.5% = 6 ÷ 2 = 3
17.5% = 12 + 6 + 3 = 21

Calculator Strategy

For more complex percentages pupils can use the following calculator methods:

e.g. Find 32% of 63.5
The unitary method in a table

100%	**63.5**	$\div$ 100
1%	0.635	X 32
32%	20.32	

Or understanding that 32% is the same as $\frac{32}{100}$ which can be written as 32 ÷ 100 to give

32 ÷ 100 x 63.5 = 20.32

Fractions

Vocabulary: Denominator (bottom), Numerator (top).

For finding quantities of an amount teach the following "divide by the bottom number, multiply by the top"

e.g. Find $\frac{4}{5}$ of 85

85 ÷ 5 = 17 → which is equivalent to $\frac{1}{5}$ of the original

17 x 4 = 68 → which is now equivalent to $\frac{4}{5}$ of the original (as it is 4 times bigger than the $\frac{1}{5}$ value just found).

Ratios

Vocabulary: Fraction, Proportion, Ratio

For finding quantities within a ratio teach the following

Sharing 4000 into the ratio 3 : 2 : 5

No. of parts = 3 + 2 + 5 = **10**
1 part is worth 4000 ÷ **10** = 400
1st → 3 x 400 = 1200
2nd → 2 x 400 = 800
3rd → 5 x 400 = 2000

You can also perform a check at this point that the 3 values you have got total the amount given in the question i.e. 1200 + 800 + 2000 = 4000.

Josh, Daniel and Thomas are left some money in the ratio 2 : 3 : 7. Josh is given £48, how much does Thomas receive?

J : D : T
2 : 3 : 7

2 parts = £48
1 part = £24
7 parts = £168, so Thomas receives £168

Order of Operations (BIDMAS or BODMAS)

Vocabulary: Brackets, Indices *plural* (Index *singular*) also know as 'Powers', Subtraction, Multiplication, Division, Addition.

Pupils are taught BIDMAS (some may know BODMAS – where the O represents the *order* or Power (index) e.g. 3^2 – the little 2 is the power) as a way to remind them which operation (+ - x or ÷ etc.) takes precedence when there are multiple parts within one calculation.

To further support the BIDMAS ordering it is actually written as

B
I
DM ↓ Complete in the order given
AS

B – Brackets	**I – Indices (powers)**	**D – Divide**	**M – Multiply**	**A – Add**	**S - Subtract**

E.g. the formula $C = \frac{f-32}{1.8}$ converts temperatures in Celsius to Fahrenheit.

If BIDMAS was not used and you wanted to find out what 100 °F was in °C you would do

C = (100 – 32) ÷ 1.8 =
68 ÷ 1.8 = 37.777…

Some pupils would put the following directly into the calculator

C = 100 - 32 ÷ 1.8 = 82.22222…

Here the pupils have continued with 'divide by 1.8' before working out the subtraction.

In the second calculation the calculator has only divided the 32 by 1.8
In the first calculation it worked out what 100 – 32 was first (68) then divided that by 1.8
Following BIDMAS would get you 37.7777… which is the correct answer as division (D) comes before subtraction (S) in the acronym BIDMAS.

Finally DM and AS is written next to each other instead of D and A
M S

Remind pupils that when calculations include these operations alone you do them in the order given in the question (i.e. without thinking about BIDMAS). If confused the below example tries to illustrate the problem about using BIDMAS in certain cases.

E.g. 10 – 5 + 3

Using	B becomes	10 – 5 + 3	
	I	10 – 8	(A before S)
	D	= 2	which is incorrect
	M		
	A		
	S		

While using	B	becomes	10 – 5 + 3	
	I		5 + 3 = 8	(order given shows the subtract before the add)
	DM			
	AS			

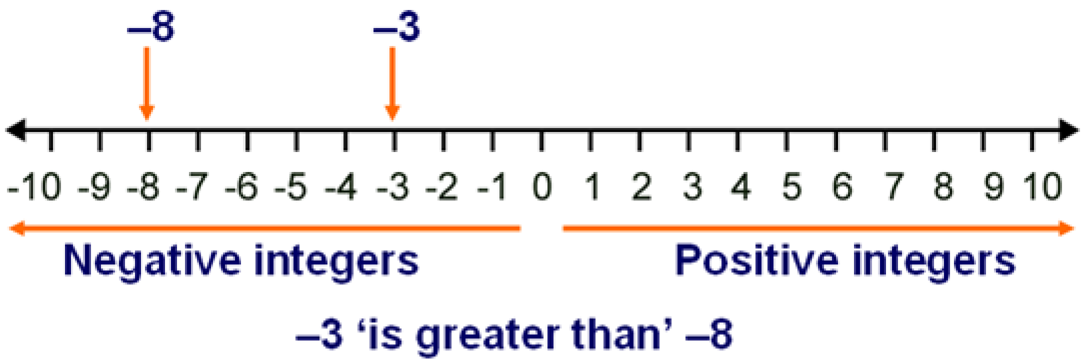
N.B. pupils often misunderstand indices

e.g. 4^2 means 4 x 4 **not** 4 x 2
and
5^3 means 5 x 5 x 5 **not** 5 x 3

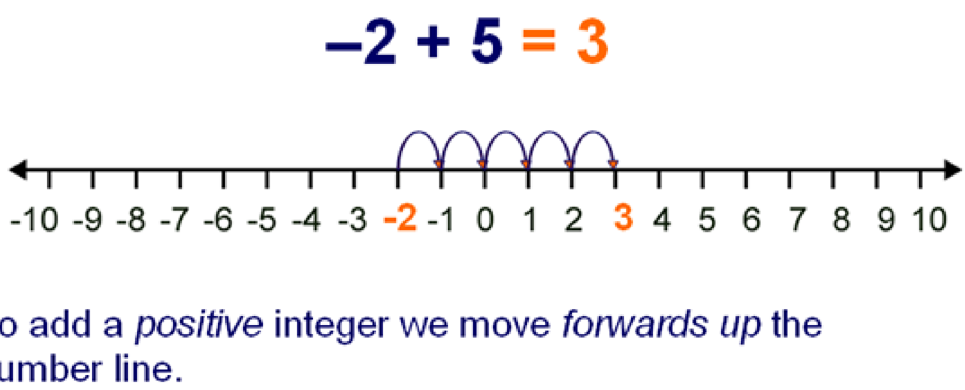
Negative (directional numbers)

<u>Vocabulary:</u> Minus, Ascending, Descending, negative, directed.

- Negative numbers are best explained via a number line
- Most subjects dealing with negatives will only need to be able to add or subtract negatives. If you need to multiply or divide with negatives please ask if unsure and follow the idea given to pupils of *two of the same makes a positive* E.g. – x – = +



Use of the number line helps pupils understand that the further to the right the **bigger** the number.



N.B. if you wish to subtract you would move *back down* (towards the negatives).

Converting between units

Vocabulary: Proportional, Unitary (used for ratios in the 1: something connection), Kilo, Centi, Milli, Metres, Grams, Miles, Inches, Feet, Pounds, Ounces, Stone, Pints, Gallons.

There are a variety of forms including metric to metric, metric to imperial (vice versa) or imperial to imperial.

The best way to aid pupils understanding is via a reverse operation ratio equivalent

X 1000 → as to make 1 become 1000 you x 1000

E.g. 1 km = 1000m

÷ 1000 → as to make 1000 become 1 you ÷ 1000

So if a question asked "Convert 2.3km to m" I'd recognise that to go from Km to m I multiply by 1000 so 2.3 x 1000 = 2300m

For conversions with more challenging numbers remind pupils that to convert 1 to any other number you multiply by that number.

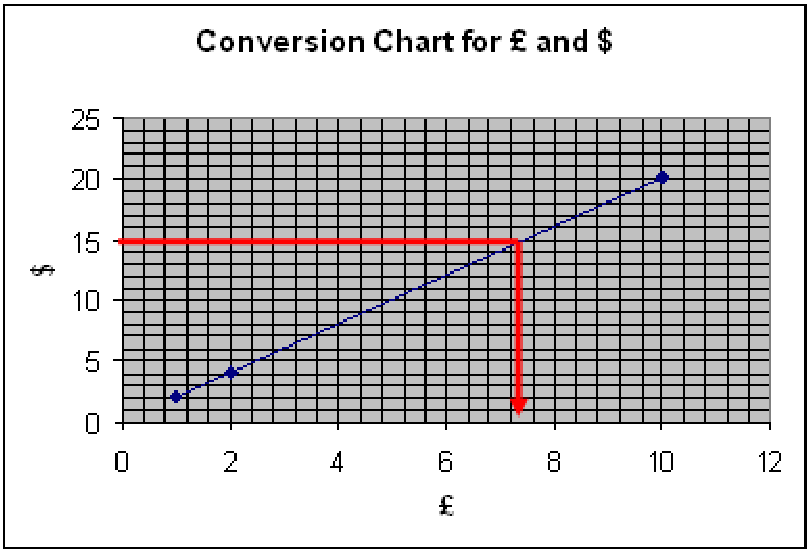
E.g. 1 kg = 2.2lbs so to make 1 become 2.2 I multiply by 2.2

Here is a list of all the common conversion facts readily used in the GCSE mathematics course.

8 Km	~	5 miles	1 Kg	~	2.2 lbs	1litre	~	1.75 pints
1 inch	~	2.5 cm	1000 Kg	=	1 metric tonne	1 gallon	=	8 pints
1 foot	~	30cm	16 oz	=	1 lb	1 litre	=	1000 cm^3
1 yard	~	90cm						

Conversion Charts

Pupils are shown the benefits of a conversion line for quick referencing.



Here $15 is around £7.30

This conversion chart can still be used for values not found on the axes.
Simply multiply a value found on the chart to find the required amount.

e.g. £6 can be used to find £600 (x100)
£6 = $12 so
£600 = $1200

Compound Measures (Speed, Density, etc)

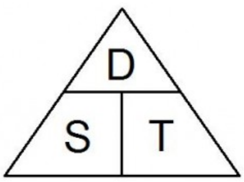
Find the speed of a cyclist who travels 50 miles in $2\frac{1}{2}$ hours.

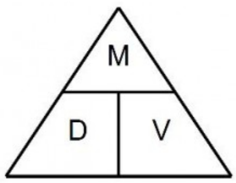
S = ? D = 50 miles T = 2.5 hours

Remind students of a speed such as 30 miles per hour.
Ask them whether 30 miles is a speed, distance or time and similarly for hour.
Write speed = distance ÷ time
Substitute values to give s = 50 ÷ 2.5 = 20 mph.

This can be linked to the family of facts for example
3 x 4 = 12
4 = 12 ÷ 3
3 = 12 ÷ 4

Other similar calculations are
Density = Mass ÷ Volume
Circumference = π × diameter $(C = \pi \times d)$
and Voltage = current x resistance $(V = I \times R)$





The speed, distance, time and density, mass, volume calculations can be solved using the triangles but it is important that pupils understand the mathematics behind the triangle.

Algebra

Vocabulary: Variable (Unknown), x^2 is "x Squared", x^3 is "x Cubed", Simplify (Collect).

Collecting like terms

When collecting like terms pupils must understand that variables cannot be mixed and numbers are also separate to variables.

e.g. $5h + 3 + 4h = 9h + 3$

Also x^2 is separate from x (collect powers separately).
e.g. $3x^2 - 5x + 6x = 3x^2 + x$ (nothing more can be collected)

NB. Some pupils need to be reminded that, with the exception of powers/indices, numbers are always before the variable. This can be likened to the English language when the number (the adjective) comes before the variable (the noun).
e.g. 9h would not be written h9 → you would describe 9h as '9 lots of h' as opposed to 'h lots of 9' in general conversation.

Solving Equations

For rearranging equations use of the balancing method is preferred

	$3x + 5 = 26$	
-5	$3x = 21$	-5
$\div 3$	$x = 7$	$\div 3$

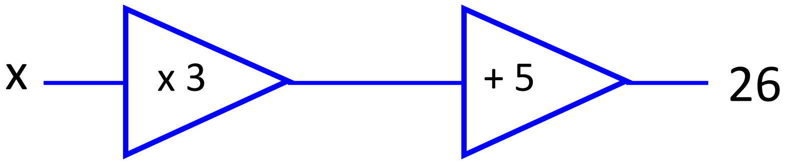
The aim here is to get the unknown (in this case x) on its own. Make sure to balance out each step, to keep the equation **equal,** by doing the same thing to **both** sides.

When pupils struggle to understand the balancing method there are two commonly used alternatives – 'Opposite on the other side' and Function Machines.

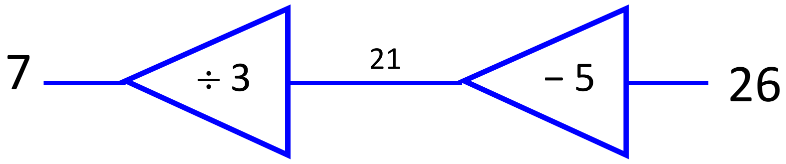
The above example would be solved as follows using the 'opposite on the other side' method.

$$3x + 5 = 26$$
$$3x = 26 - 5$$
$$3x = 21$$
$$x = 21 \div 3$$
$$x = 7$$

With function machines we would represent the problem as follows.



Then reversing the arrows and performing the opposite functions gives



Expanding brackets

Vocabulary: Expand (simplify)

When expanding brackets use of the box method can be used

$2(x – 5)$

	x	-5
2	2x	-10

$= 2x – 10$

However it is more common to use arrows as follows:

2(x – 5) = '2 times x' – '2 times 5'

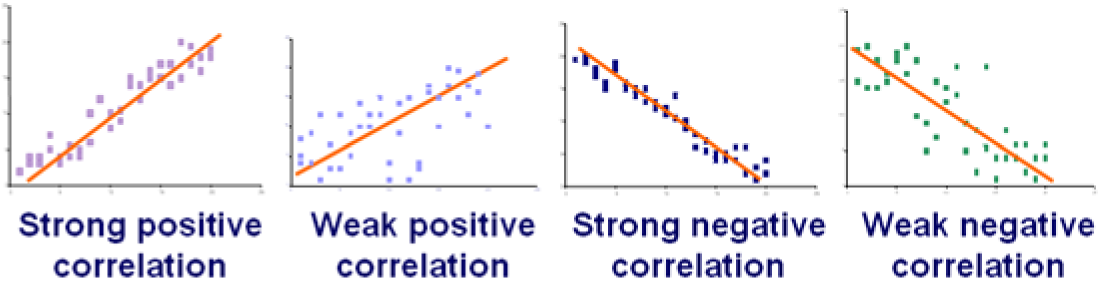
= 2x – 10

Handling Data

Scatter Graphs

<u>Vocabulary:</u> Correlation, Positive, Negative, Anomalous, Mean, Line of Best Fit, Gradient (slope), Axes *plural* (Axis *singular*), Origin (0,0), outlier.

- For certain subjects (some sciences) scatter graphs can be drawn not as discussed below although mathematically speaking the following is the preferred approach.



It should be noted that scatter graphs try to connect two types of data. If the points are all randomly spread out across the graph with no trend then we say there is **no correlation**. It may also be noted that the two sets of data may be placed either way round on the axes.

When plotting your own scatter graph it is not needed at KS3 in mathematics to ask the pupils to plot a *line of best fit* (see the orange lines in the above examples) until Year 8 for all pupils.

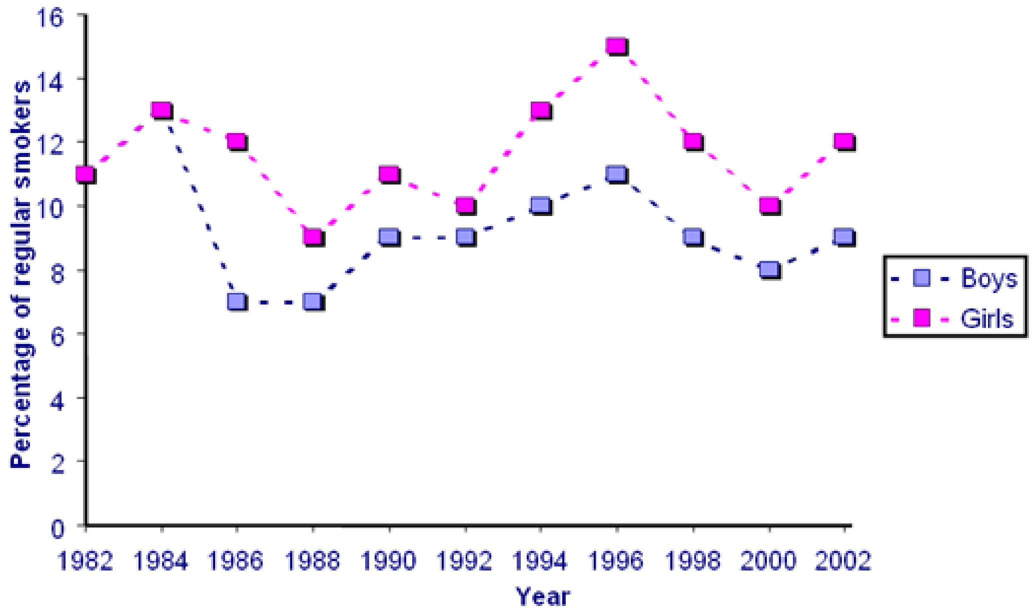
It is also not needed in the majority of classes (until KS4) to plot the line of best fit with the ***mean point*** which is used to stabilise the *line of best fit*. The mean point is found by adding all the values then dividing by the number of values and doing this for two separate sets of data to find the mean point co-ordinates.

In some Sciences Scatter Graphs can be curved. All Scatter Graphs used within mathematics will however be straight.

Line and Bar Graphs

<u>Vocabulary:</u> Frequency, Axes *plural* (Axis *singular*), Polygon, Co-ordinates, Key, Labels, Trend, Continuous, Discrete, Mid point, Class Width, Vertical, Horizontal, Scale.

- Line graphs are mathematically known as *Frequency Polygons.* They are used to show **trend** for continuous*[1] data. If a line graph is used to highlight discrete*[2] data a dotted line rather than a solid line should be used. Some pupils confuse these with *Bar Line Graphs* (Bar Charts that use thin lines instead of a thick bar).
-

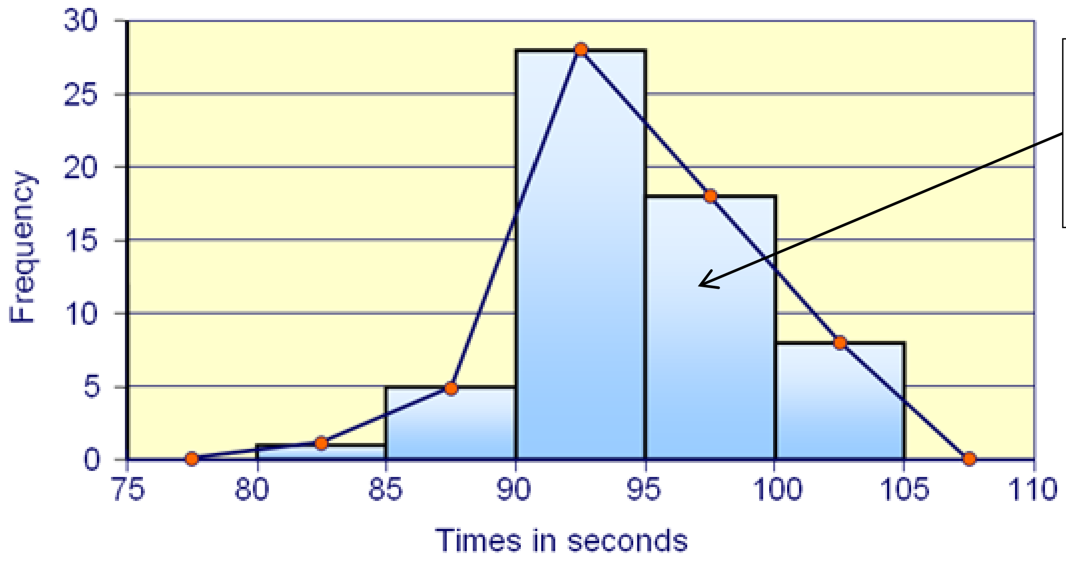


It should be noted here that as the lines going between the years do not actually mean anything a dotted line is used to simply show the **trend**.

[1] ***Continuous*** data is data that you can measure and can take any value e.g. distance, time, weight etc.

[2] ***Discrete*** data is data you can count, e.g. number of brothers, favourite colour …

Frequency Polygon to show the number of Pupils completing a race.



Rub out bars once finished with.

This illustrates that a Frequency Polygon can be drawn by understanding how to draw a bar chart. The points of the Frequency Polygon are the centre positions of the bars. These centre points are referred to as the *mid points*.

N.B. the Frequency Polygon is the straight lines alone, the bars are not needed.

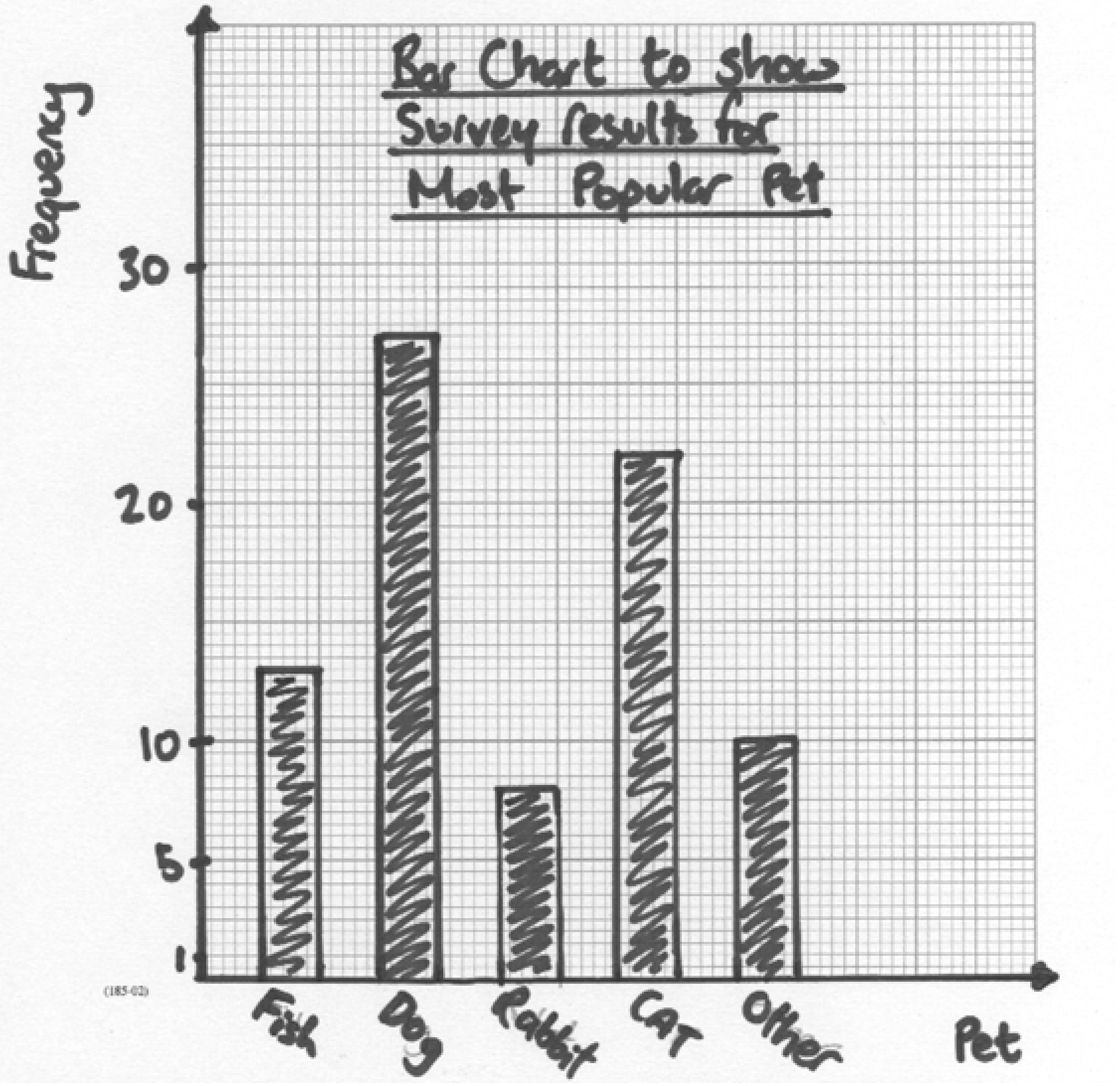
Frequency diagrams (Bar Charts)

Gaps between the bars are used for *discrete* data.

Gary carries out a survey to find the most popular type of pet.
The results of his survey are shown in the following table.

Type of pet	Tally	Frequency
Fish	𝍸 𝍸 ///	13
Dog	𝍸 𝍸 𝍸 𝍸 𝍸 //	27
Rabbit	𝍸 ///	8
Cat	𝍸 𝍸 𝍸 𝍸 //	22
Other	𝍸 𝍸	10

Complete the frequency column in the above table and use this data to draw a suitable bar chart on the graph paper below. [4]



No gaps are used for *continuous* data.

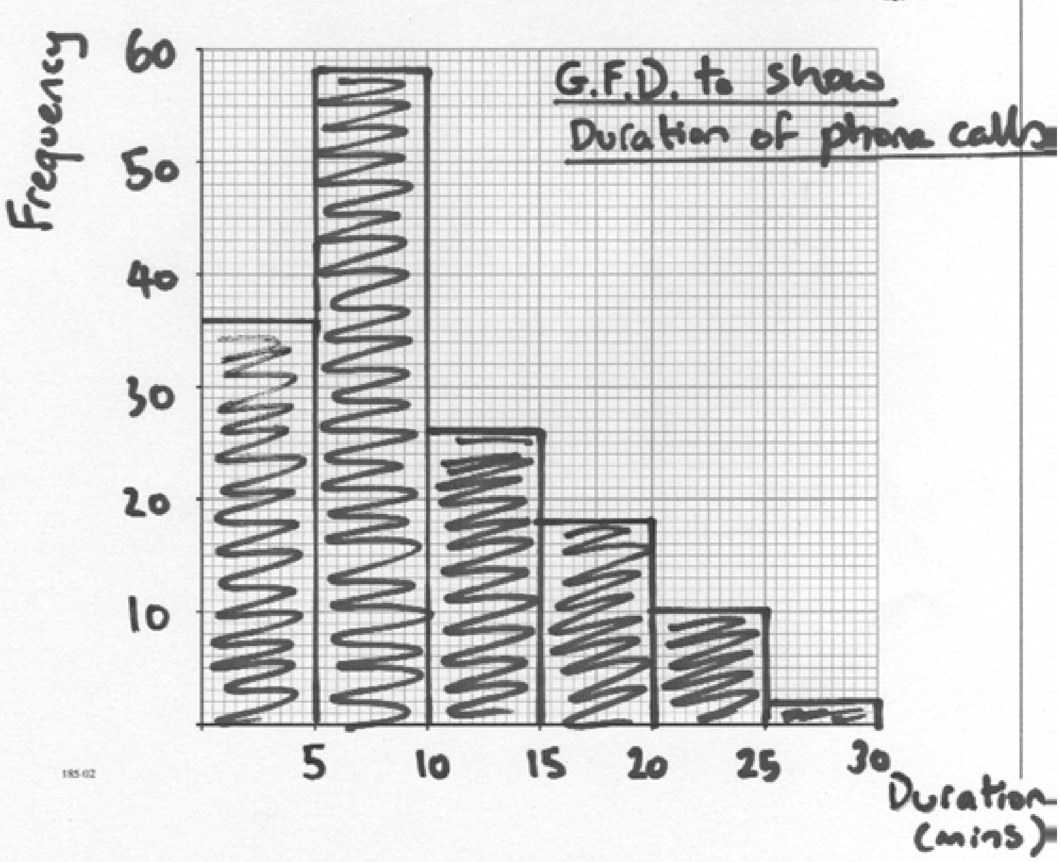
The duration, in minutes, of each of 150 phone calls was recorded. The table shows a grouped frequency distribution of the results.

Duration of phone call in minutes (t)	Number of phone calls
$0 < t \leqslant 5$	36
$5 < t \leqslant 10$	58
$10 < t \leqslant 15$	26
$15 < t \leqslant 20$	18
$20 < t \leqslant 25$	10
$25 < t \leqslant 30$	2

Here the < and ≤ symbols mean greater than 5 and up to (and including) 10.

(a) On the graph paper below, draw a grouped frequency diagram for the data.

= Bar Chart [3]

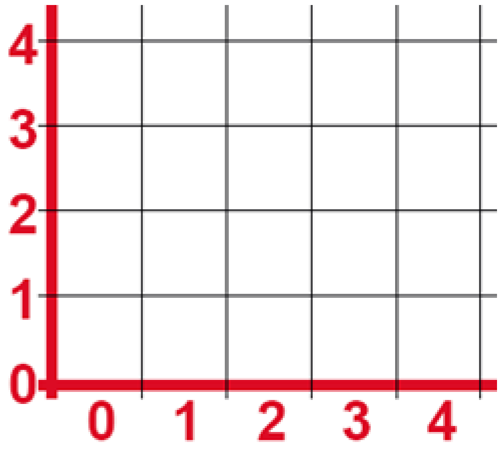


N.B.

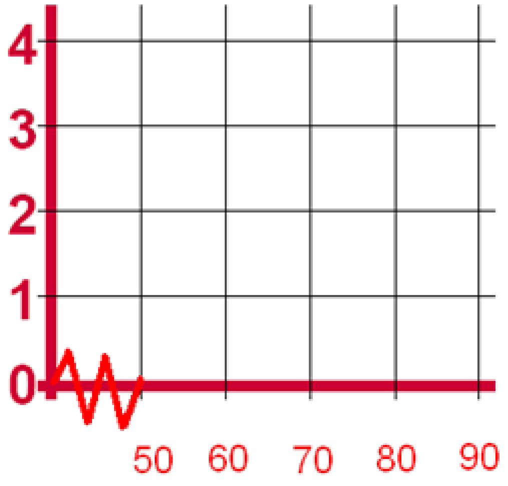
1. Please note that continuous data bar charts must always have the bottom axes of numbers spread out (not grouped).
2. A Grouped Frequency Diagram is the name of a histogram based on groups.

Things to look out for…

1. Placing the numbers in the centre of the squares (which is **only** acceptable if the data is discrete).



2. Starting on one, or both of the axes on a number higher than 0. The zigzag line is used to denote that values previous to the ones noted on that axis are not needed.



3. Pupils often make the mistake of having inconsistent scales on their axes.

Pie Charts

Vocabulary: Sector, Frequency, Angle, Sum, Fraction, Labels, Radius, Protractor.

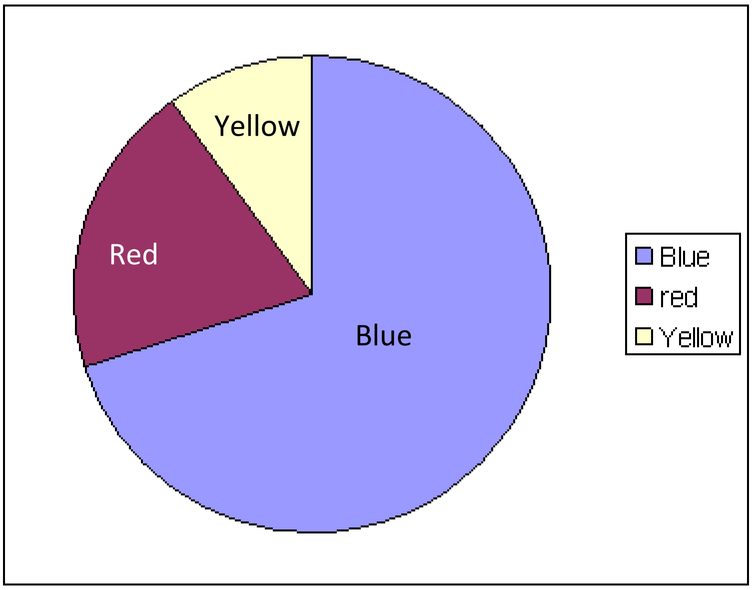
When calculating the angles of each sector (the slice or part of the pie chart for each option) using a fractional method it is a good idea to create a table.

e.g.

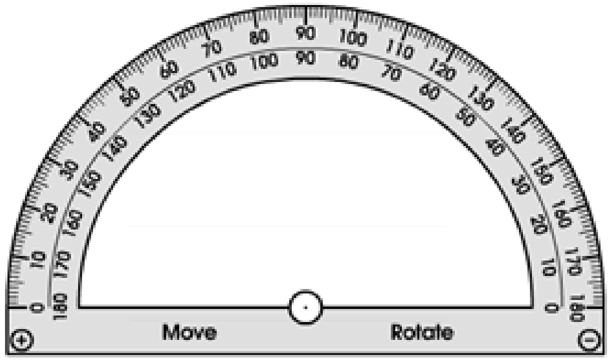
Favourite colour	**Frequency**	**Workings**	**Angle**
Blue	7	36 x 7	252
Red	2	36 x 2	72
Yellow	1	36 x 1	32

Total = 10
So 1 person is represented by an angle of $360^{o} \div 10 = 36^{o}$.
Therefore, blue (in the above example) has 7 responses so the total blue angle is 7 lots of 36.

If a pupil needs support with pie charts ask them how they find a fraction of an amount.



Either type of labelling is fine
A) Using a key
B) Name each sector



When getting the pupils to measure the sectors' angles ask them to …

1. Estimate the size of the angle.
2. Start the protractor with zero degrees at the NORTH position.
3. Turn the protractor clockwise to complete the other sectors.

e.g. in above example the 1st angle = 225° which is more than a straight line. Which means when drawn the sector shouldn't look less than a straight line.

When the pupils are measuring reflex (more than 180° angles) get them to make the 180° point **then** turn the protractor and measure the remaining 45° (225 - 180).

When the pupils measure this 45° again ask them to *estimate* the angle, in this case less than 90° (a quarter turn). If they do this they will not get confused about which 45° marking on the protractor they should use.

However, pupils also commonly use full circle protractors like the one below. With these they must be encourage to line the line they are drawing or measuring an angle from along the 0° line on the protractor and then follow the values ascending from 0°.

Averages

<u>Vocabulary:</u> The mean is sometimes referred to as the "average". Mean, Mode, Median, Range, Data, Frequency, Sum, Total, Quantitative Data.

- There are 3 averages mean, mode, median
- Range is often used with the averages to aid analysis of the *spread of data*. This idea is linked to the understanding of consistency.

i.e. the smaller the range the more consistent the data set.

To help the pupils remember what the different averages are and to remind them how to find them a song sung to the tune of Frere Jacque.

♫ <u>Average Song</u> ♫
Mean is Average
Mean is Average
Mode is Most
Mode is Most
Median is the middle
Median is the middle
Range High Low
Range High Low

Finding the Mean

(often just referred to as the Average outside of mathematics)

<u>From a list of Values</u>

Add the data then divide the total number of pieces of data.

E.g. 5,4,5,3,3

5+4+5+3+3= 20 → 20/5 = 4

From a Frequency Table

Find the subtotals for each different value then find the sum of these. It is important to note here that a common mistake is that pupils divide by the number of values (groups) rather than the number of pieces of data (the total frequency).

Shoe Size	1	2	3	4	5	Total
Frequency	2	3	4	1	0	10
Shoe Size x Frequency	2	6	12	4	0	24

$$\text{mean} = \frac{24}{10} = 2.4$$

Finding the mean from grouped data

For this you'll need to use the mid point. The mean in this case is called an **Estimated Mean**. The reason why it is an estimate is due to the fact that the mid point value for each group is used. This is because the actual values are not known and to restrict the amount of error the middle value is used.

Height (cm)	Mid Point	Frequency	Totals
$110 < h \le 130$	120	4	480
$130 < h \le 150$	140	3	420
$150 < h \le 170$	160	3	480
	Totals	10	1380

$$\text{Estimated mean} = \frac{\text{Sum of Totals}}{\text{Total Frequency}} = 138 \text{ cm}$$

Finding the Median

The median is the middle value of a set of data **after** the data has been put in ascending (or descending) order. If there are two values in the middle we find the mean of those (add them then divide by 2).

E.g. 3, 3, 4, 5, 6, 7, 7

some pupils find it easier to cross off numbers, one from the front and one from the back until they reach the middle.

In another example, if the data was 3, 4, 5, 6

There would be 2 middle values. Here we must add these then divide by 2 to find the median. So 4 + 5 = 9 → 9/2 = 4.5

Finding the Mode

(also referred to as the **Modal value**)

The mode is the most popular value. The mode is the only average that can be found of qualitative (non – numerical) data.

E.g. Favourite colour
Green, Green, Blue, Red, Black.

Here you can state that Green is the most popular choice.
So the **Mode = Green**
You couldn't however add the colours up and divide to calculate the mean, or for that matter, put them in order and find the middle for the median.

Finding the Range

The range **is not** an average but is used with averages to form an analysis of them. It is a single value found by subtracting the smallest from the largest value. Pupils need to be made aware that

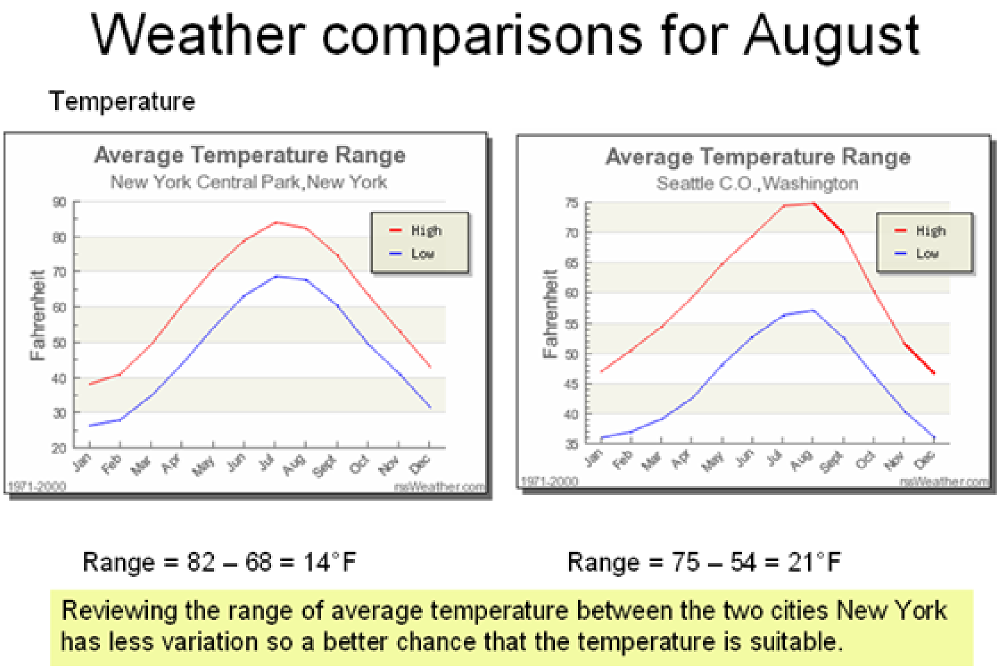
*the smaller the range the more **consistent** the data*

and

the larger the range the more unreliable or larger the variation in the results.

NB – in Science they refer to the Range as the lowest value to the highest value or vice versa and **not** as a single value.

e.g.



Questionnaires

<u>Vocabulary:</u> Questionnaire.

There are some basic guidelines that can be applied to questionnaires that ensure they are fit for purpose.

- Don't make the questionnaire too long
    - *6 to 8 questions will usually suffice*

- Ensure that questions are concise and are not open to interpretation – give time periods if necessary
    - *How many bars of chocolate do you eat **per week**?*

- Don't ask embarrassing, sensitive or irrelevant questions
    - *For example, you wouldn't ask someone for their weight or salary*

- Don't ask leading or biased questions
    - *A poor question would be, "Normal people think One Direction are the best group in the world, do you agree?"*

- Give response boxes that are exhaustive (i.e. cover all possibilities) and do not overlap
    - For example, how many DVD's do you buy per year? (tick one box)

0	1-3	4-6	7- 9	10 or more
☐	☐	☐	☐	☐

Shape

Area

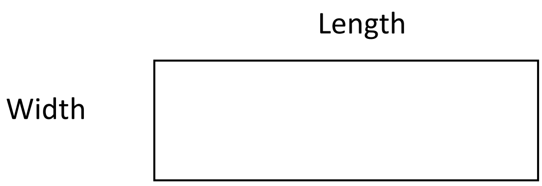
Vocabulary: Length, Width, Height, Radius, Pi (π), Squared, Perpendicular, Milli, Centi, Kilo, Metres.

Here is a list of the formulae that pupils will meet within Key Stage 3.

Pupils are firstly introduced to area as the amount of surface covered inside a shape's boundary. There are two approaches taught, counting squares (shapes drawn on squared paper) or via the corresponding formula.

Rectangle, Square, Parallelogram, Rhombus

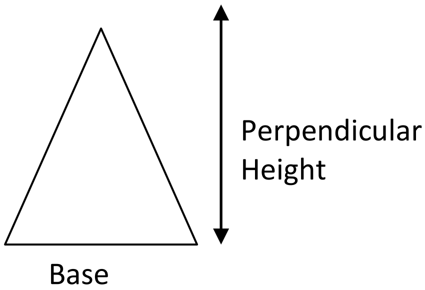
Area = Length x Width (measurements must be perpendicular to each other)



A shape's "length" is generally considered as the longest distance, although it makes no difference to the final outcome.

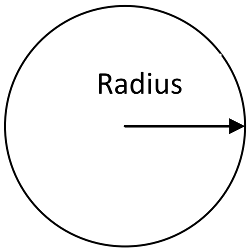
Triangle

Area = base x height ÷ 2 (measurements must be perpendicular to each other)



Circle

Area = πr^2 = π x radius x radius

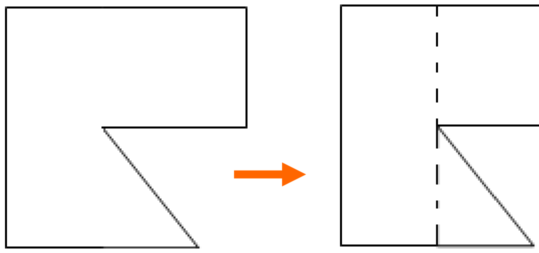


Pupils can have issues over the formula for the area of a circle when quoted as "pi r squared" (r^2) rather than the "radius x radius" approach (as given above). This is due to the fact that they will often believe that squared is the same as multiplying by 2. Therefore the use of the formula as written above allows for less confusion about this fact.

Composite Shapes

Pupils are taught to look for smaller known shapes that can be combined to produce the overall composite shape given.

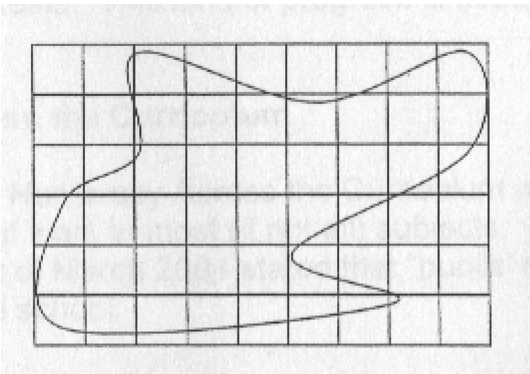
E.g.
Here we have a composite shape made from a rectangle, a square and a triangle.



One option

Estimating the area of irregular shapes

Use a grid background and count the squares. For part used squares common sense should be used, connect multiple part-squares that would roughly make full squares. Pupils should be told to cross the counted squares off or use a methodical approach to prevent miscounting.

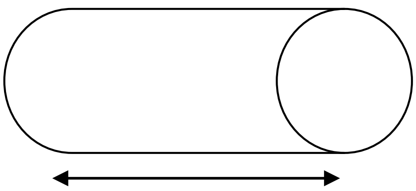


Volume

Vocabulary: Length, Width, Height, Radius, Pi (π), Cubed (cm^3), Perpendicular, Milli, Centi, Kilo, Metres, Plane, Face, Cross-section, Area, Prism.

Pupils are introduced to the volume of a prism.

Volume of a Prism = Area of Cross-Section x Length



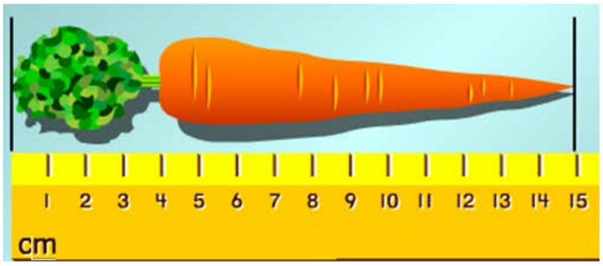
Length

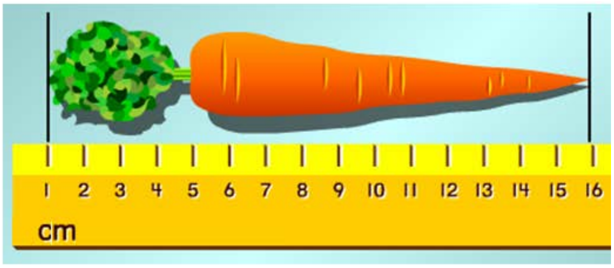
In this case the cross-sectional face of this prism is a circle. So the area of a circle formula is used in the above formula.

Measuring and reading scales

<u>Vocabulary:</u> Length, Milli, Centi, Kilo, Metre, Gram, Inch, Yard, Mile, Ounce, Pounds, Degrees, Protractor, Ruler, Scale, Clockwise, Acute, Obtuse, Reflex, Right-Angle.

The Mathematics department measure in cm as a default however pupils are taught mm too. The main difficulties with measuring will occur when the scale is not going up by one, as the case may be with weighing scales etc.





Here you notice a correct and an incorrect use of a ruler to measure the carrot. Some pupils get confused and start measuring from 1cm rather than 0cm.

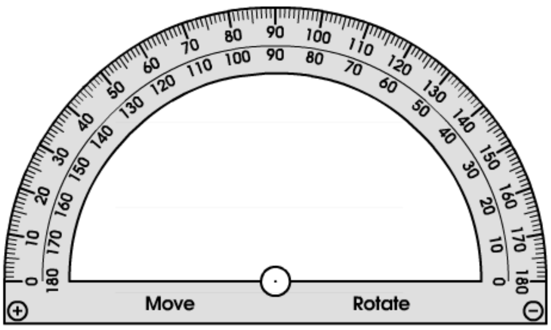
If the scales are awkward get the pupils to work out the value of one section of the scale before attempting any actual measurements. Calculators could be used for some classes.

1. Here the bottom metric scale has two types of lines separations. For the main ones → 2 gaps and an increase of 0.2 (1 → 1.2) so each interval = 0.2 ÷ 2 = 0.1

2. For the smaller intervals → 5 gaps and an increase of 0.1 so each interval is = 0.1 ÷ 5 = 0.02

<u>Angles and Protractors</u>

Recap on using protractors for reflex angles…



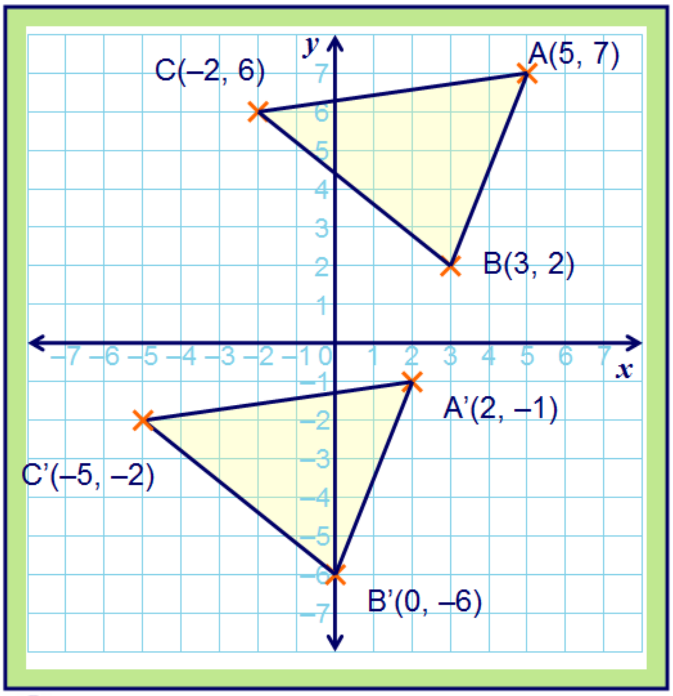
Measuring angles larger than 180° (reflex angles)
To measure 225° get them to make the 180° point **then** turn the protractor and measure the remaining 45° (225 - 180).

When the pupils measure this 45° again ask them to *estimate* the angle, in this case less than 90° (a quarter turn). If they do this they will not get confused about which 45° marking on the protractor they should use.

Remind pupils to always check they are measuring an angle by starting from the "track" (2 tracks run in opposite directions) that starts with 0 rather than 180!

Co-ordinates

Vocabulary: Axis (pl. Axes), X, Y, Co-ordinate



There are a variety of phrases the pupils know to remember the order of direction that co-ordinates state. The most used are

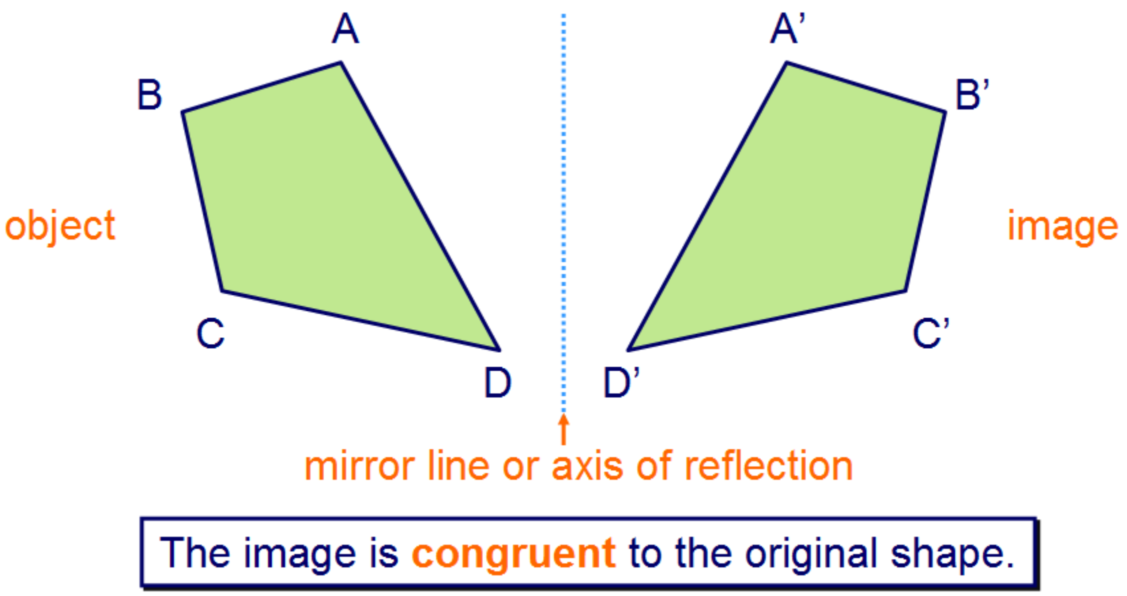
"Across the landing up the stairs."
"Along the corridor up the stairs."
"x comes before y in the alphabet."

i.e. along the x axis then along the y axis.

Transformations

<u>Vocabulary:</u> Reflection, Rotation, Enlargement, Centre, Object, Image, Congruent, Vertex (pl. Vertices), Symmetry, Corresponding.

Reflection



Notice for reflection that the "object" is the original and the "image" is the new position. Congruent refers to the new position being exactly the same size and shape as the original. A^1 refers to the new position of A.

If pupils have difficulty with reflections, especially diagonal lines and cross overs, tracing paper can be used. The pupils draw around the object and also mark the position of the mirror line. Then they flip the paper over and line up the mirror line to then have the new position for the image. Another approach is to encourage the pupils to turn the problem so that the mirror line is either vertical or horizontal and then complete the reflection.

Rotation

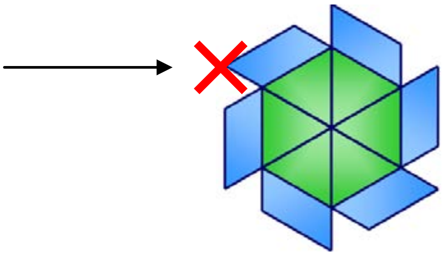
The pupils will need to consider direction, amount of turn and from what position, known as the centre of rotation.

Turn is taught in both degrees and fractions

¼ = 90° ½ = 180° ¾ = 270°

Tracing paper is always used for rotation. Pupils find the method very straightforward. The only thing to remind them is to mark a vertex on the shape so they remember which part of the shape they started with.

Start is



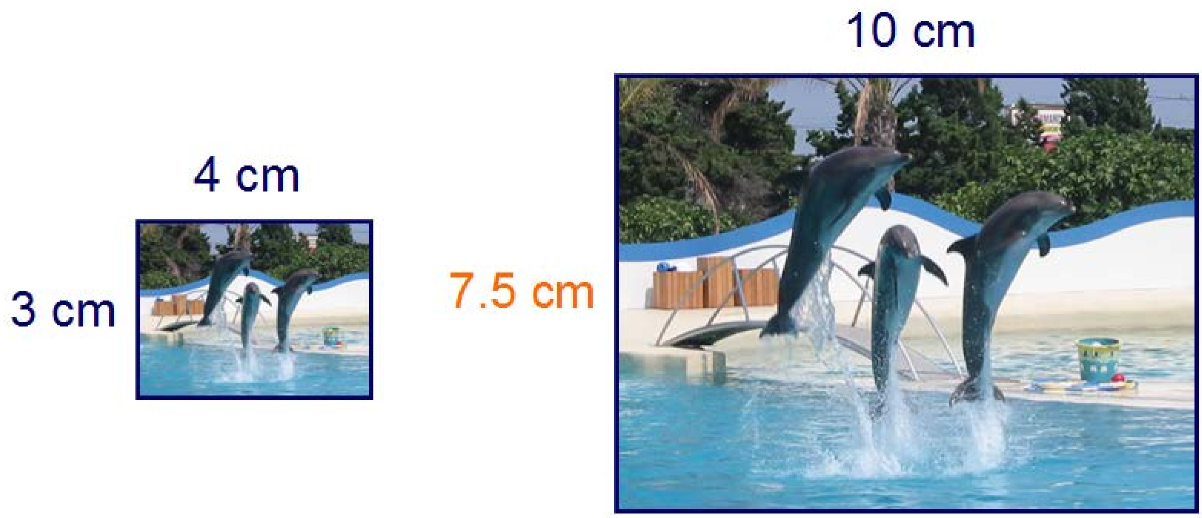
When describing the amount of rotational symmetry an object has the phrase is "Order … rotational symmetry"

e.g. above has ORDER 6 ROTATIONAL SYMMETRY as it would fit over itself 6 times during a complete turn.

However it should be noted that all objects with ORDER 1 ROTATIONAL SYMMETRY are actually referred to as having "NO ROTATIONAL SYMMETRY" as any object will fit over itself as least once.

Enlargement

The pupils will need to consider the amount of increase, known as the SCALE FACTOR. More capable pupils will also be taught to enlarge a shape from a certain position, known as the CENTRE OF ENLARGEMENT, and even fractional and negative scale factors (ask the Mathematics Department if you require any clarification with these types of enlargement).
Here is the object and image of a photo of dolphins. Once the scale factor is known the pupils must remember to **multiply** all distances of the object by the scale factor to find the new distances for the image.



For harder questions the pupils may be required to find out the scale factor. In this case the following approach is taught…

NEW MEASUREMENT ÷ OLD MEASUREMENT = SCALE FACTOR

N.B. the measurements quoted in this formula are corresponding lengths, which mean they are the equivalent versions of each other.

E.g., 10 ÷ 4 = 2.5 ← the scale factor.

The mathematics used for enlargement is the same mathematics used for direct proportion, scales on maps, currency exchange and any area of conversion where you have a relationship that has zero equal zero. For example if you change £0 you will get $0 and if this was graphed it would be a straight line through the origin. The scale factor actually gives the gradient of the graph. The idea of proportion is a mathematical concept that underpins a wide range of taught mathematics.