# 2.1 AS UNIT 1

**Unit 1: Pure Mathematics A**
Written examination: 2 hours 30 minutes
25% of A level qualification (62.5% of AS qualification)
120 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

| Topics | Guidance |
|---|---|
| **2.1.1 Proof** | |
| Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including<br>(a) proof by deduction,<br>(b) proof by exhaustion,<br>(c) disproof by counter example. | Proof by deduction to include the proofs of the laws of logarithms. |
| **2.1.2 Algebra and Functions** | |
| Understand and use the laws of indices for all rational exponents.<br>Use and manipulate surds, including rationalising the denominator. | To include rationalising fractions such as $\frac{2+3\sqrt{5}}{3-2\sqrt{5}}$ and $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$. |
| Work with quadratic functions and their graphs.<br>The discriminant of a quadratic function, including the conditions for real roots and repeated roots.<br><br>Completing the square.<br><br>Solution of quadratic equations in a function of the unknown. | The nature of the roots of a quadratic equation.<br><br><br><br>To include finding the maximum or minimum value of a quadratic function.<br>To include by factorisation, use of the formula and completing the square. |

| Topics | Guidance |
|---|---|
| Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation. | To include finding the points of intersection or the point of contact of a line and a curve. |
| Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.<br><br>Express solutions through the correct use of 'and' and 'or', or through set notation.<br><br>Represent linear and quadratic inequalities graphically. | To include the solution of inequalities such as $1 - 2x < 4x + 7$, $\frac{x}{2} \geq 2(1-3x)$ and $x^2 - 6x + 8 \geq 0$.<br><br>To include, for example, $y > x+1$ (a strict inequality) and $y \geq ax^2 + bx + c$ (a non-strict inequality). |
| Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem. | The use of the Factor Theorem will be restricted to cubic polynomials and the solution of cubic equations. |
| Understand and use graphs of functions; sketch curves defined by simple equations, including polynomials.<br>$y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, including their vertical and horizontal asymptotes.<br><br>Interpret algebraic solutions of equations graphically.<br>Use intersection points of graphs of curves to solve equations.<br><br>Understand and use proportional relationships and their graphs. | The equations will be restricted to the form $y = f(x)$. |
| Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs:<br>$y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$. | |

| Topics | Guidance |
|---|---|
| **2.1.3 Coordinate geometry in the ($x$, $y$) plane** | |
| Understand and use the equation of a straight line, including the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.<br><br>Be able to use straight line models in a variety of contexts. | To include<br>• finding the gradient, equation, length and midpoint of a line joining two given points;<br>• the equations of lines which are parallel or perpendicular to a given line. |
| Understand and use the coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle.<br><br>Use of the following circle properties:<br>(i) the angle in a semicircle is a right angle;<br>(ii) the perpendicular from the centre to a chord bisects the chord;<br>(iii) the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point. | To also be familiar with the equation of a circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.<br><br>To include:<br>• finding the equations of tangents,<br>• the condition for two circles to touch internally or externally,<br>• finding the points of intersection or the point of contact of a line and a circle, |
| **2.1.4 Sequences and Series - The Binomial Theorem** | |
| Understand and use the binomial expansion of $(a + bx)^n$ for positive integer $n$ .<br>The notations $n!$, $\binom{n}{r}$ and $n\text{C}r$.<br><br>Link to binomial probabilities. | To include use of Pascal's triangle. |

| Topics | Guidance |
|---|---|
| **2.1.5 Trigonometry** | |
| Understand and use the definitions of sine, cosine and tangent for all arguments. | Use of the exact values of the sine, cosine and tangent of 30°, 45° and 60°. |
| Understand and use the sine and cosine rules, and the area of a triangle in the form ½ $ab\sin C$. | To include the use of the sine rule in the ambiguous case. |
| Understand and use the sine, cosine and tangent functions.<br>Understand and use their graphs, symmetries and periodicity. | |
| Understand and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.<br>Understand and use $\cos^2\theta + \sin^2\theta = 1$. | These identities may be used to solve trigonometric equations or prove trigonometric identities. |
| Solve simple trigonometric equations in a given interval, including quadratic equations in $\sin$, $\cos$ and $\tan$, and equations involving multiples of the unknown angle. | To include the solution of equations such as<br>$3\sin\theta = 1$, $\tan\theta = \dfrac{\sqrt{3}}{2}$, $3\cos 2\theta = -1$ and $2\cos^2\theta + \sin\theta - 1 = 0$. |
| **2.1.6 Exponentials and logarithms** | |
| Know and use the function $a^x$ and its graph, where $a$ is positive.<br><br>Know and use the function $e^x$ and its graph. | |
| Know that the gradient of $e^{kx}$ is equal to $ke^{kx}$ and hence understand why the exponential model is suitable in many applications. | Realise that when the rate of change is proportional to the $y$ value, an exponential model should be used. |

| Topics | Guidance |
|---|---|
| Know and use the definition of $\log_a x$ as the inverse of $a^x$, where $a$ is positive and $x \geq 0$.<br><br>Know and use the function $\ln x$ and its graph.<br>Know and use $\ln x$ as the inverse function of $e^x$. | |
| Understand and use the laws of logarithms.<br>$\log_a x + \log_a y = \log_a (xy)$<br><br>$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$<br><br>$k \log_a x = \log_a (x^k)$ (including, for example $k = -1$, $k = -½$) | To include the proof of the laws of logarithms.<br>Use of the laws of logarithms.<br>e.g. Simplify $\log_2 36 - 2\log_2 15 + \log_2 100 + 1$.<br><br>Change of base will not be required. |
| Solve equations in the form $a^x = b$. | The use of a calculator to solve equations such as<br>(i) $3^x = 2$,<br>(ii) $25^x - 4 \times 5^x + 3 = 0$.<br>(iii) $4^{2x+1} = 5^x$ |
| Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for $x$ and $y$. | Link to laws of logarithms.<br>Understand that on a graph of $\log y$ against $\log x$, the gradient is $n$ and the intercept is $\log a$, and that on a graph of $\log y$ against $x$, the gradient is $\log b$ and the intercept is $\log k$. |
| Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as model for population growth.)<br><br>Consideration of limitations and refinements of exponential models. | The formal differentiation and integration of formulae involving $e^x$ and/or $a^x$ will not be required. |

| Topics | Guidance |
|---|---|
| **2.1.7 Differentiation** | |
| Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point $(x, y)$; the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second order derivatives.<br><br>Differentiation from first principles for small positive integer powers of $x$.<br><br>Understand and use the second derivative as the rate of change of gradient. | The notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $f'(x)$ may be used.<br><br><br><br>Up to and including power of 3.<br>To include polynomials up to and including a maximum degree of 3. |
| Differentiate $x^n$ for rational $n$, and related constant multiples, sums and differences. | To include polynomials. |
| Apply differentiation to find gradients, tangents and normals, maxima and minima, and stationary points.<br>Identify where functions are increasing or decreasing. | To include finding the equations of tangents and normals.<br>The use of maxima and minima in simple optimisation problems.<br>To include simple curve sketching. |
| **2.1.8 Integration** | |
| Know and use the Fundamental Theorem of Calculus. | Integration as the reverse of differentiation. |
| Integrate $x^n$ (excluding $n = -1$) and related sums, differences and constant multiples. | To include polynomials. |
| Evaluate definite integrals.<br>Use a definite integral to find the area under a curve. | To include finding the area of a region between a straight line and a curve. |

| Topics | Guidance |
|---|---|
| **2.1.9 Vectors** | |
| Use vectors in two dimensions. | To include the use of the unit vectors, **i** and **j**. |
| Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations. | Condition for two vectors to be parallel. |
| Understand and use position vectors; calculate the distance between points represented by position vectors.<br><br>Use vectors to solve problems in pure mathematics. | Use of **AB** = **b** – **a**.<br>To include the use of position vectors given in terms of unit vectors.<br><br>To include the use and derivation of the position vector of a point dividing a line in a given ratio. |

# 2.2 AS UNIT 2

**Unit 2: Applied Mathematics A**
Written examination: 1 hour 45 minutes
15% of A level qualification (37.5% of AS qualification)
75 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1.

The paper will comprise two sections:

**Section A: Statistics (40 marks)**

**Section B: Mechanics (35 marks)**

The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as candidates deem appropriate.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

| Topics | Guidance |
|---|---|
| **STATISTICS** | |
| **2.2.1 Statistical Sampling** | |
| Understand and use the terms 'population' and 'sample'.<br>Use samples to make informal inferences about the population.<br><br>Understand and use sampling techniques, including simple random sampling, systematic sampling and opportunity sampling. | |
| Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population. | |

| Topics | Guidance |
|---|---|
| **2.2.2 Data presentation and interpretation** | |
| Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency.<br><br>Connect to probability distributions. | Learners should be familiar with box and whisker diagrams and cumulative frequency diagrams.<br>Qualitative assessment of skewness is expected and the use of the terms symmetric, positive skew or negative skew |
| Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population.<br><br>(Calculations of coefficients of regression lines are excluded.)<br><br>Understand informal interpretation of correlation.<br><br>Understand that correlation does not imply causation. | Equations of regression lines may be given in a question and learners asked to make predictions using it.<br><br>Use of the terms positive, negative, zero, strong and weak is expected. |
| Interpret measures of central tendency and variation, extending to standard deviation.<br><br>Be able to calculate standard deviation, including from summary statistics. | Measures of central tendency: mean, median, mode.<br>Measures of central variation: variance, standard deviation, range, interquartile range. |
| Recognise and interpret possible outliers in data sets and statistical diagrams.<br><br>Select or critique data presentation techniques in the context of a statistical problem.<br><br>Be able to clean data, including dealing with missing data, errors and outliers. | Use of $Q_1 - 1.5 \times IQR$ and $Q_3 + 1.5 \times IQR$ to identify outliers. |

| Topics | Guidance |
|---|---|
| **2.2.3 Probability** | |
| Understand and use mutually exclusive and independent events when calculating probabilities.<br><br>Link to discrete and continuous distributions. | To include the multiplication law for independent events:<br>$P(A \cap B) = P(A)\,P(B)$. |
| Use Venn diagrams to calculate probabilities. | Use of set notation and associated language is expected.<br><br>To include the generalised addition law:<br>$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.<br><br>Conditional probability will not be assessed in this unit. |
| **2.2.4 Statistical distributions** | |
| Understand and use simple, discrete probability distributions.<br><br>Understand and use,<br>• the binomial distribution, as a model<br>• the Poisson distribution, as a model<br>• the discrete uniform distribution, as a model<br><br>(Calculation of mean and variance of discrete random variables is excluded.) | To include using distributions to model real world situations and to comment on their appropriateness. |
| Calculate probabilities using<br><br>• the binomial distribution.<br><br>• the Poisson distribution.<br><br>• the discrete uniform distribution. | Use of the binomial formula and tables / calculator.<br><br>Use of the Poisson formula and tables / calculator<br><br>Use of the formula for the discrete uniform distribution. |

| Topics | Guidance |
|---|---|
| Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial, Poisson or discrete uniform model may not be appropriate. | |
| **2.2.5 Statistical hypothesis testing** | |
| Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value. | The *p*-value is the probability that the observed result or a more extreme one will occur under the null hypothesis $H_0$.<br>For uniformity, interpretations of a *p*-value should be along the following lines:<br>$p < 0{\cdot}01$; there is very strong evidence for rejecting $H_0$.<br>$0{\cdot}01 \leq p \leq 0{\cdot}05$; there is strong evidence for rejecting $H_0$.<br>$p > 0{\cdot}05$; there is insufficient evidence for rejecting $H_0$. |
| Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.<br><br>Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis. | |
| Interpret and calculate Type I and Type II errors, and know their practical meaning. | |

| Topics | Guidance |
|---|---|
| **MECHANICS** | |
| **2.2.6 Quantities and units in mechanics** | |
| Understand and use fundamental quantities and units in the S.I. system; length, time and mass. | |
| Understand and use derived quantities and units: velocity, acceleration, force, weight. | |
| **2.2.7 Kinematics** | |
| Understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration. | |
| Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of the gradient; velocity against time and interpretation of the gradient and the area under the graph | Learners may be expected to sketch displacement-time and velocity-time graphs. |
| Understand, use and derive the formulae for constant acceleration for motion in a straight line. | To include vertical motion under gravity.<br>Gravitational acceleration, $g$.<br><br>The inverse square law for gravitation is not required and $g$ may be assumed to be constant, but learners should be aware that $g$ is not a universal constant but depends on location.<br>The value 9·8 $\text{ms}^{-2}$ can be used for the acceleration due to gravity, unless explicitly stated otherwise. |
| Use calculus in kinematics for motion in a straight line. | To include the use of<br>$v = \frac{\mathrm{d}r}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$, $r = \int v\,\mathrm{d}t$, $v = \int a\,\mathrm{d}t$, where $v$, $a$ and $r$ are given in terms of $t$. |

| Topics | Guidance |
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| **2.2.8 Forces and Newton's laws** | |
| Understand the concept of a force.<br>Understand and use Newton's first law. | |
| Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors). | |
| Understand and use weight and motion in a straight line under gravity; gravitational acceleration, $g$, and its value in S.I. units to varying degrees of accuracy.<br><br>(The inverse square law for gravitation is not required and $g$ may be assumed to be constant, but learners should be aware that $g$ is not a universal constant but depends on location.) | Forces will be constant and will include weight, normal reaction, tension and thrust.<br>To include problems involving lifts.<br><br>The value 9·8 $ms^{-2}$ can be used for the acceleration due to gravity, unless explicitly stated otherwise. |
| Understand and use Newton's third law.<br>Equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors)<br><br>Applications to problems involving smooth pulleys and connected particles. | Problems involving particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be<br>(i) freely hanging,<br>(ii) on a smooth, horizontal plane. |
| **2.2.9 Vectors** | |
| Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form. | |
| Use vectors to solve problems in context, including forces. | Does not include kinematics problems. |

# 2.3 A2 UNIT 3

**Unit 3: Pure Mathematics B**
Written examination : 2 hours 30 minutes
35% of A level qualification
120 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

| Topics | Guidance |
|---|---|
| **2.3.1 Proof** | |
| Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs). | |
| **2.3.2 Algebra and Functions** | |
| Simplify rational expressions, including by factorising and cancelling and by algebraic division (by linear expressions only). | |
| Sketch curves defined by the modulus of a linear function. | Be able sketch graphs of the form $y = \|ax + b\|$.<br>To include solving equations and inequalities involving the modulus function. |
| Understand and use composite functions; inverse functions and their graphs. | Understand and use the definition of a function.<br>Understand and use the domain and range of functions.<br><br>In the case of a function defined by a formula (with unspecified domain) the domain is taken to be the largest set such that the formula gives a unique image for each element of the set.<br><br>The notation *fg* will be used for composition. |

| Topics | Guidance |
|---|---|
| Understand the effect of combinations of transformations on the graph of $y = f(x)$, as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$. | |
| Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear). | With denominators of the form $(ax + b)(cx + d)$, $(ax+b)(cx+d)(ex+f)$ and $(ax + b)(cx + d)^2$. Learners will **not** be expected to sketch the graphs of rational functions. |
| Use of functions in modelling, including consideration of limitations and refinements of the models. | |
| **2.3.3 Coordinate geometry in the ($x$, $y$) plane** | |
| Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms. | To include finding the equations of tangents and normals to curves defined parametrically or implicitly.<br>Knowledge of the properties of curves other than the circle will **not** be expected. |
| Use parametric equations in modelling in a variety of contexts. | |

| Topics | Guidance |
|---|---|
| **2.3.4 Sequences and Series** | |
| Understand and use the binomial expansion of $(a+bx)^n$, for any rational $n$, including its use for approximation.<br><br>Be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\|<1$ (proof not required). | To include the expansion, in ascending powers of $x$, of expressions such as $(2-x)^{\frac{1}{2}}$ and $\frac{(4-x)^{\frac{3}{2}}}{(1+2x)}$. |
| Work with sequences, including those given by a formula for the $n$th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.<br>Increasing sequences, decreasing sequences, periodic sequences. | |
| Understand and use sigma notation for sums of series. | |
| Understand and work with arithmetic sequences and series, including the formulae for the $n$th term and the sum to $n$ terms. | Use of $u_n = a+(n-1)d$.<br><br>Use and proof of $S_n = \frac{n}{2}[2a+(n-1)d]$ and $S_n = \frac{n}{2}[a+l]$. |
| Understand and work with geometric sequences and series, including the formulae for the $n$th term and the sum of a finite geometric series.<br><br>The sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation. | Use of $u_n = ar^{n-1}$.<br><br>Use and proof of $S_n = \frac{a(1-r^n)}{1-r}$.<br><br>Use of $S_\infty = \frac{a}{1-r}$ for $\|r\| < 1$. |
| Use sequences and series in modelling. | |

| Topics | Guidance |
|---|---|
| **2.3.5 Trigonometry** | |
| Work with radian measure, including use for arc length, area of sector and area of segment. | |
| Understand and use the standard small angle approximations of sine, cosine and tangent.<br><br>$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{\theta^2}{2}$ and $\tan\theta \approx \theta$, where $\theta$ is in radians. | |
| Know and use exact values of $\sin$ and $\cos$ for 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, $\pi$ and multiples thereof, and exact values of tan for 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, $\pi$ and multiples thereof. | |
| Understand and use the definitions of sec, cosec, cot, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Understand the relationships of all of these to sin, cos and tan and understand their graphs, ranges and domains. | |
| Understand and use $\sec^2\theta \equiv 1+\tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1+\cot^2\theta$. | The solution of trigonometric equations such as $\sec^2\theta + 5 = 5\tan\theta$. |
| Understand and use double angle formulae.<br>Use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$.<br>Understand geometric proofs of these formulae. | Use of these formulae to solve equations in a given range, e.g. $\sin 2\theta = \sin\theta$,<br>Applications to integration, e.g. $\int \cos^2 x \, dx$. |

| Topics | Guidance |
|---|---|
| Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$. | Use of these to solve equations in a given range, e.g. $3\cos\theta + \sin\theta = 2$.<br>Application to finding greatest and least values, e.g. the least value of $\frac{1}{3\cos\theta + 4\sin\theta + 10}$. |
| Construct proofs involving trigonometric functions and identities. | |
| **2.3.6 Differentiation** | |
| Differentiation from first principles for $\sin x$ and $\cos x$. | |
| Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves, and points of inflection. | Points of inflection to include stationary and non-stationary points. |
| Differentiate $e^{kx}$, $a^{kx}$, $\sin kx$, $\cos kx$, $\tan kx$, and related sums, differences and constant multiples.<br><br>Understand and use the derivative of $\ln x$. | |
| Apply differentiation to find points of inflection. | |
| Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions. | To include the use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$. |
| Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only. | |
| Construct simple differential equations in pure mathematics. | |

| **2.3.7 Integration** | |
|---|---|
| Integrate $e^{kx}$, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples. | Use of the results:<br>1) if $\int f(x)dx = F(x) + k$ then $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c$.<br>2) $\int f'(g(x))g'(x)dx = f(g(x)) + c$ |
| Use a definite integral to find the area between two curves. | |
| Understand and use integration as the limit of a sum. | |
| Carry out simple cases of integration by substitution and integration by parts.<br>Understand these methods as the reverse processes of the chain rule and the product rule respectively.<br><br>Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated.<br><br>Integration by parts includes more than one application of the method but excludes reduction formulae. | |
| Integrate using partial fractions that are linear in the denominator. | |
| Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.<br>(Separation of variables may require factorisation involving a common factor.) | Questions will be set in pure mathematics only. |

| **2.3.8 Numerical Methods** | |
|---|---|
| Locate roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of $x$ in which $f(x)$ is sufficiently well-behaved.<br>Understand how change of sign methods can fail. | |
| Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams.<br><br>Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$.<br><br>Understand how such methods can fail. | The iterative formula will be given.<br>Consideration of the conditions for convergence will **not** be required. |
| Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between. | Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether it gives an overestimate or an underestimate of the area under a curve.<br><br>Simpson's rule is excluded. |
| Use numerical methods to solve problems in context. | To solve problems in context which lead to equations that cannot be solved analytically. |

# 2.4 A2 UNIT 4

**Unit 4: Applied Mathematics B**
Written examination: 1 hour 45 minutes
25% of A level qualification
80 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1, Unit 2 and Unit 3.

The paper will comprise two sections:

**Section A: Statistics (40 marks)**

**Section B: Differential Equations and Mechanics (40 marks)**

The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as candidates deem appropriate.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

| Topics | Guidance |
|---|---|
| **STATISTICS** | |
| **2.4.1 Probability** | |
| Understand and use conditional probability, including the use of tree diagrams, Venn diagrams and two-way tables. | |
| Understand and use the conditional probability formula: $P(A \cap B) = P(A)\,P(B\mid A) = P(B)\,P(A\mid B)$. | |
| Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions. | |

| Topics | Guidance |
|---|---|
| **2.4.2 Statistical distributions** | |
| Understand and use the continuous uniform distribution and Normal distributions as models.<br><br>Find probabilities using the Normal distribution.<br><br>Link to histograms, mean, standard deviation, points of inflection and the binomial distribution. | Use of calculator / tables to find probabilities.<br>Linear interpolation in tables will **not** be required. |
| Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the continuous uniform or Normal model may not be appropriate. | The distributions from which the selection can be made are:<br>Discrete: binomial, Poisson, uniform<br>Continuous: Normal, uniform |
| **2.4.3 Statistical hypothesis testing** | |
| Understand and apply statistical hypothesis testing to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given *p*-value or critical value.<br><br>(The calculation of correlation coefficients is excluded.) | Learners will be expected to state hypotheses in terms of $\rho$, where $\rho$ represents the population correlation coefficient. |
| Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance, and interpret the results in context. | Learners should know and be able to use the result that<br>if $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$<br>(The proof is excluded.) |

| Topics | Guidance |
|---|---|
| **DIFFERENTIAL EQUATIONS AND MECHANICS** | |
| **2.4.4 Trigonometry** | |
| Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces. | Contexts may include, for example, wave motion as well as problems in vector form which involve resolving directions and quantities in mechanics. |
| **2.4.5 Differentiation** | |
| Construct simple differential equations in context (contexts may include kinematics, population growth and modelling the relationship between price and demand). | To include contexts involving exponential growth and decay. |
| **2.4.6 Integration** | |
| Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. | Questions will be set in context.<br>Separation of variables may require factorisation involving a common factor. |
| Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics. | |

| Topics | Guidance |
|---|---|
| **2.4.7 Quantities and units in mechanics** | |
| Understand and use derived quantities and units for moments. | |
| **2.4.8 Kinematics** | |
| Extend, use and derive the formulae for constant acceleration for motion in a straight line to 2 dimensions using vectors. | |
| Extend the use of calculus in kinematics for motion in a straight line to 2 dimensions using vectors. | To include the use of $\mathbf{v}=\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$, $\mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}$, $\mathbf{r}=\int\mathbf{v}\,\mathrm{d}t$, $\mathbf{v}=\int\mathbf{a}\,\mathrm{d}t$, where $\mathbf{v}$, $\mathbf{a}$ and $\mathbf{r}$ are given in terms of $t$. |
| Model motion under gravity in a vertical plane using vectors; projectiles. | To include finding the speed and direction of motion of the projectile at any point on its path.<br>The maximum horizontal range of a projectile for a given speed of projection.<br>In examination questions, learners may be expected to derive the general form of the formulae for the range, the time of flight, the greatest height or the equation of path.<br>In questions where derivation of formulae has not been requested, the quoting of these formulae will **not** gain full credit.<br><br>Questions will **not** involve resistive forces. |

| Topics | Guidance |
|---|---|
| **2.4.9 Forces and Newton's laws** | |
| Extend Newton's second law to situations where forces need to be resolved (restricted to two dimensions). | |
| Resolve forces in two dimensions.<br>Understand and use the equilibrium of a particle under coplanar forces. | |
| Understand and use addition of forces; resultant forces; dynamics for motion in a plane. | |
| Understand and use the $\mathrm{F} \leq \mu \mathrm{R}$ model for friction.<br>The coefficient of friction.<br>The motion of a body on a rough surface.<br>Limiting friction and statics. | Forces will be constant and will include weight, friction, normal reaction, tension and thrust.<br>To include motion on an inclined plane.<br>The motion of particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be on an inclined plane. |
| **2.4.10 Moments** | |
| Understand and use moments in simple static contexts. | To include parallel forces only. |
| **2.4.11 Vectors** | |
| Understand and use vectors in three dimensions. | To include the use of the unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. |
| Use vectors to solve problems in context, including forces and kinematics. | Questions will not involve the scalar product. |